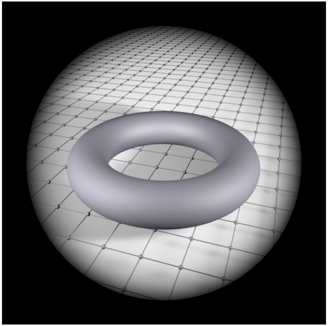
Creating and Probing Topological Matter with Cold Atoms: From Shaken Lattices to Synthetic Dimensions

Nathan Goldman



2015 Arnold Sommerfeld School, August–September 2015

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Outline

Part 1: Shaking atoms!

- Generating effective Hamiltonians: “Floquet” engineering
- Topological matter by shaking atoms
- Some final remarks about energy scales

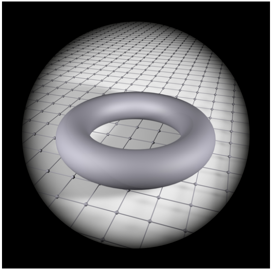
Part 2: Seeing topology in the lab!

- Loading atoms into topological bands
- Anomalous velocity and Chern-number measurements
- Seeing topological edge states with atoms

Part 3: Using internal atomic states!

- Cold Atoms = moving 2-level systems
- Internal states in optical lattices: laser-induced tunneling
- Synthetic dimensions: From 2D to 4D quantum Hall effects

Part 2: Seeing topology in the lab!



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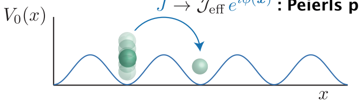
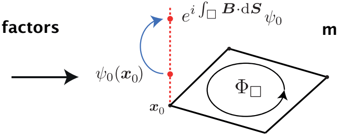
Designing topological models by shaking atoms

- The **basic concept**:

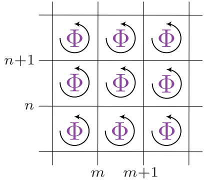
$J \to \mathcal{J}_{\rm eff}\, e^{i\phi(\boldsymbol{x})}$ **: Peierls phase-factors**





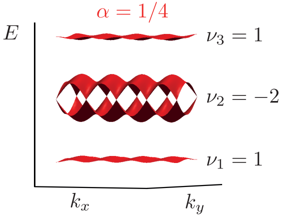
mimic the A-B effect

- Munich/MIT: The **Harper-Hofstadter** model

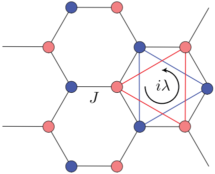


$$\hat{H} = -J \sum_{m,n} \hat{a}^{\dagger}_{m,n+1}\hat{a}_{m,n} + e^{i2\pi\alpha n}\hat{a}^{\dagger}_{m+1,n}\hat{a}_{m,n} + \text{H.c.}$$

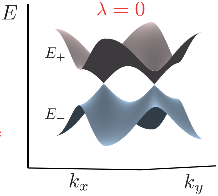
$\alpha = \Phi/\Phi_0$: uniform flux per plaquette (in units of flux quantum)

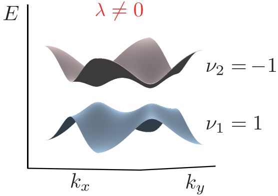


- ETH Zurich: The **Haldane** model

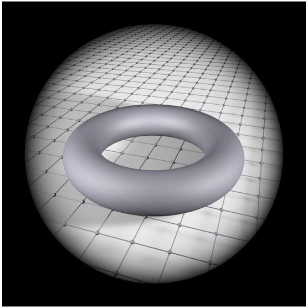


$$\hat{H} = -J \sum_{\langle j,k \rangle} \hat{a}^{\dagger}_{j}\hat{a}_{k} + \lambda \sum_{\langle\langle j,k \rangle\rangle} i^{\circlearrowleft}\hat{a}^{\dagger}_{j}\hat{a}_{k}$$

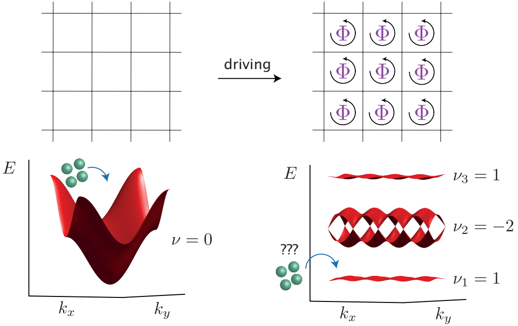




Loading atoms into topological bands



- Starting from a 2D optical square lattice : $E(\boldsymbol{k}) = -2J\left[\cos(k_x d) + \cos(k_y d)\right]$



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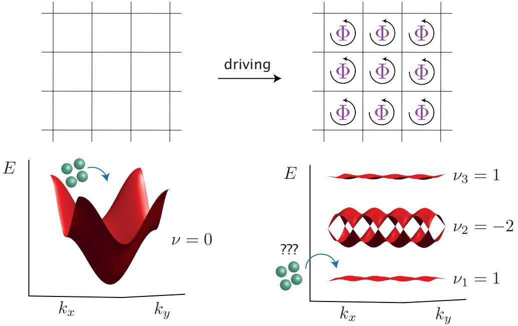
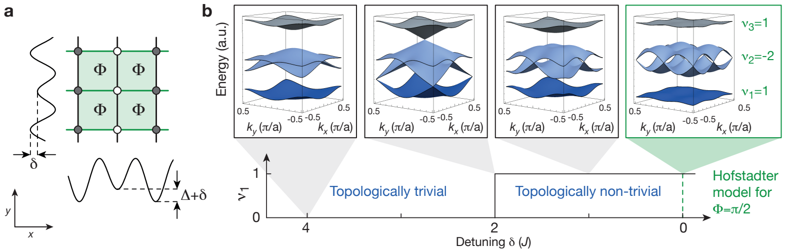
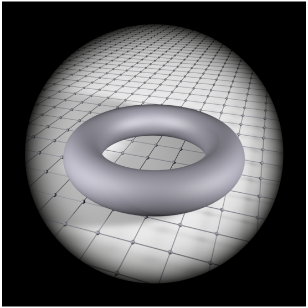


- The Munich trick [Aidelsburger et al. Nature Phys. ’15] : Nb of bands preserved !

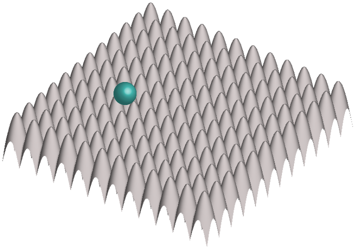


Anomalous velocity
and Chern-number measurements



The Berry curvature in a lattice system

- Consider a particle moving on a two-dimensional lattice:

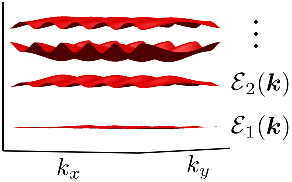


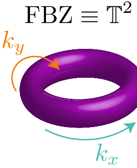
- The eigenfunctions are Bloch waves

$$\psi_{n,\boldsymbol{k}}(\boldsymbol{r}) = e^{i\boldsymbol{r}\cdot\boldsymbol{k}} u_{n,\boldsymbol{k}}(\boldsymbol{r})$$

- The eigenenergies are Bloch bands

$$\hat{H}_{\boldsymbol{k}}\, u_{n,\boldsymbol{k}} = \mathcal{E}_n(\boldsymbol{k})\, u_{n,\boldsymbol{k}}$$





- The **Berry curvature** of the n**th band**: $\Omega_n = \frac{1}{2}\Omega_n^{\mu\nu} \mathrm{d}k_\mu \wedge \mathrm{d}k_\nu = \Omega_n^{xy} \mathrm{d}k_x \wedge \mathrm{d}k_y$

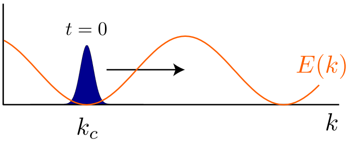
$$\Omega_n^{xy} = i\left[\langle \partial_{k_x} u_n | \partial_{k_y} u_n \rangle - \langle \partial_{k_y} u_n | \partial_{k_x} u_n \rangle\right]$$

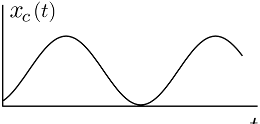
Topology of the nth Bloch band: $\frac{1}{2\pi}\int_{\mathbb{T}^2} \Omega_n = \nu_n \in \mathbb{Z}$ **: Chern number of the band**

Bloch Oscillations and the Anomalous (Berry) Velocity

- Consider a particle moving on a **1D** lattice **and subjected to a constant force** F
- The semi-classical equations of motion for a **wave packet** centered around x_c and k_c in a Bloch band $E(k)$

$$\begin{cases} \dfrac{\mathrm{d}x_c(t)}{\mathrm{d}t} = \dfrac{\partial E(k)}{\hbar \partial k} = v_{\text{band}} \\ k_c(t) = Ft/\hbar \end{cases}$$





Bloch oscillations

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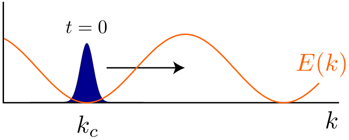
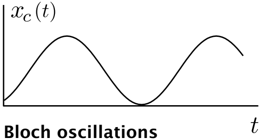




- Consider a particle moving on a **2D** lattice **and subjected to a constant force** $\boldsymbol{F} = F_y \mathbf{1}_y$
- The averaged velocity in a state $u_{n,\boldsymbol{k}}$ is given by:

$$v_n^x(\boldsymbol{k}) = \frac{\partial E_n(\boldsymbol{k})}{\hbar \partial k_x} - \frac{F_y}{\hbar}\, \Omega_n^{xy}(\boldsymbol{k}) \quad \textbf{: anomalous (Berry) velocity}$$

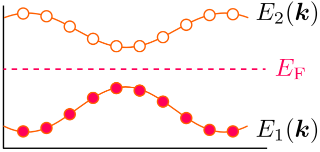
Ref: Karplus & Luttinger 1954

$$v_n^y(\boldsymbol{k}) = \frac{\partial E_n(\boldsymbol{k})}{\hbar \partial k_y}$$

Isolate the Berry velocity? Populate all the states in nth band: $\sum_{\mathbf{k}} v_{\text{band}}^{x,y}(\boldsymbol{k}) \longrightarrow \int_{\mathbb{T}^2} v_{\text{band}}^{x,y}\, \mathrm{d}^2 k = 0$

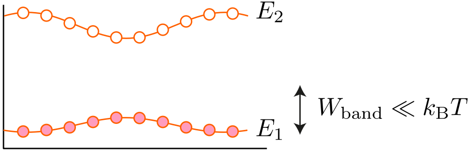
Isolating the Berry velocity: Uniformly populating a single band

- **Filled band of fermions**



$\rho = N_\mathrm{part}/N_\mathrm{states} = 1$

- **Thermal gas**



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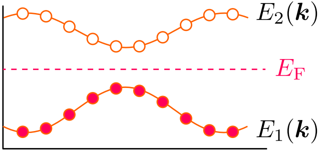

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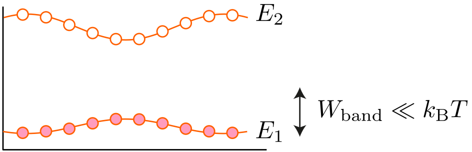

$\rho = N_{\text{part}}/N_{\text{states}} \neq 1$

- Let us compute the transverse velocity: $v_{\text{tot}}^x = -\dfrac{F_y}{h} N_{\text{part}} A_{\text{cell}}\, \nu_1$ where $\nu_1 = \dfrac{1}{2\pi} \int_{\mathbb{T}^2} \Omega_1^{xy}(\boldsymbol{k}) \mathrm{d}^2 k$

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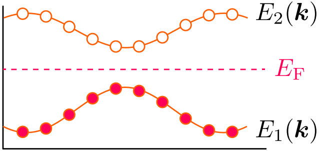

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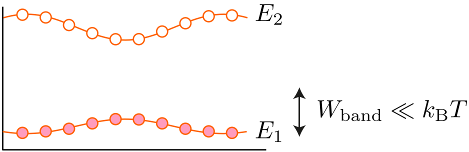

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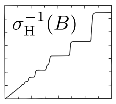
- Link with the electrical Hall conductivity: $j_x = \sigma_{xy} E_y$ where $j_x = e v^x_\mathrm{tot}/A_\mathrm{syst}$ and $E_y = F_y/e$

$$\sigma_\mathrm{H} = -\sigma_{xy} = \frac{e^2}{h}\rho\,\nu_1$$

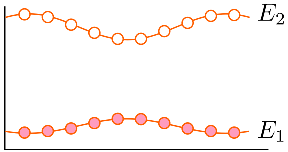
Filled bands of fermions: $$\sigma_\mathrm{H} = \frac{e^2}{h}\sum_{\text{filled band } n} \nu_n$$ ⟶ integer quantum Hall effect

von Klitzing 1980



The Thermal Bose Gas and the Center-of-Mass Drift

- **Thermal Bose gas**



$W_{\text{band}} \ll k_{\text{B}}T$

- The filling factor: $\rho = N_{\text{part}}/N_{\text{states}} \neq 1$
- The Hall conductivity: $\sigma_{\text{H}} = \frac{e^2}{h}\rho\,\nu_1$ where $j_x = \sigma_{\text{H}}\,F_y$

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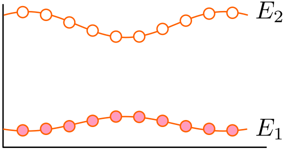

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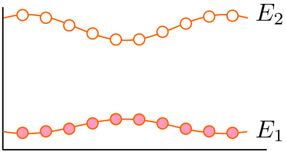

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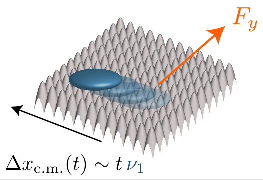
- The transverse velocity: $v^x_{\rm tot} = -\frac{F_y}{h} N_{\rm part} A_{\rm cell}\,\nu_1$

- The **center-of-mass** transverse velocity: $v^x_{\rm c.m.} = v^x_{\rm tot}/N_{\rm part} = -\frac{F_y A_{\rm cell}}{h}\nu_1 \longrightarrow \not\rho$

- The center-of-mass drift: $\Delta x_{\rm c.m.}(t) = -\frac{F_y A_{\rm cell}}{h} t\,\nu_1$ see A. Dauphin & NG PRL 2013

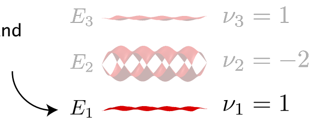
In-situ imaging can reveal the Chern number ν_1

Measure the Chern number with ultracold bosons

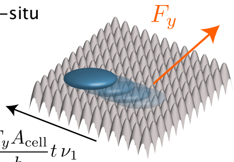


The Chern–number experiment in Munich

- The bosons are loaded into the lowest band

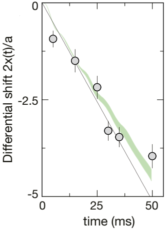


- The optical gradient is added and the transverse drift is imaged in–situ



$$\Delta x_{\mathrm{c.m.}}(t) = -\frac{F_y A_{\mathrm{cell}}}{h} t\, \nu_1$$

- Experimental data: $x(t, \Phi) - x(t, -\Phi) = 2x(t)$



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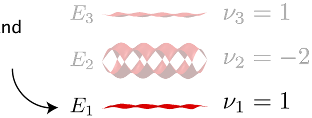

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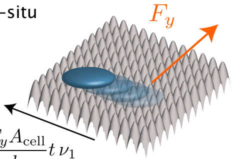

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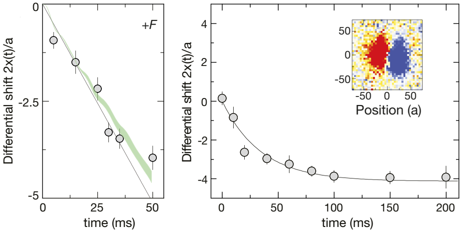

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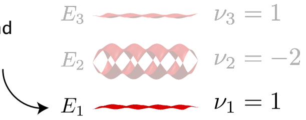

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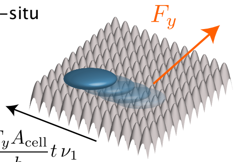

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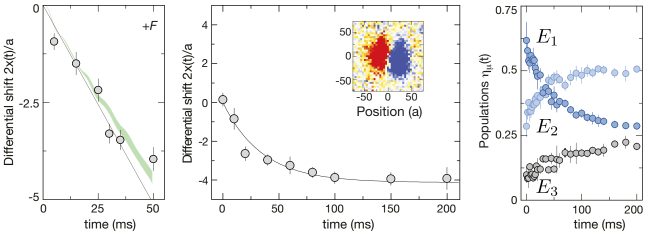

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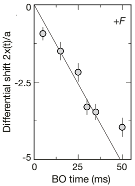
Analyzing the data

- Short-time analysis: taking into account real **initial band populations** (about **60% in lowest band**)

$$\Delta x_{\mathrm{c.m}}(t) = -\frac{F_y A_{\mathrm{cell}}}{h} t \left\{ \nu_1 \eta_1 + \nu_2 \frac{\eta_2}{2} + \nu_3 \eta_3 \right\} = -\frac{F_y A_{\mathrm{cell}}}{h} t \; \nu_1 \gamma_0$$



where $\gamma_0 = \eta_1 - \eta_2 + \eta_3$

+ band-mapping data $\eta^0_{1,2,3} = \{0.55(6), 0.31(3), 0.13(3)\}$ $\longrightarrow$ $\nu_{\mathrm{exp}} = 0.9(2)$

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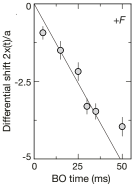
where $\gamma_0 = \eta_1 - \eta_2 + \eta_3$



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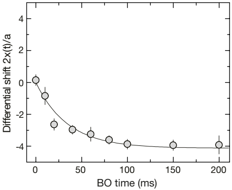
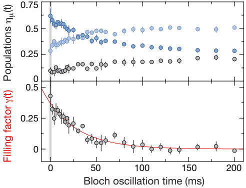
- Long-time analysis: taking into account **band repopulation**

$$\Delta x_{\rm c.m}(t) = -\frac{F_y A_{\rm cell}}{h} \nu_1 \int_0^t \gamma(t')\mathrm{d}t \quad \text{where} \quad \gamma(t) = \eta_1(t) - \eta_2(t) + \eta_3(t)$$



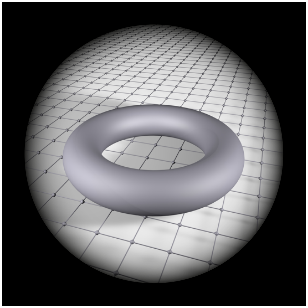
+ band-mapping data



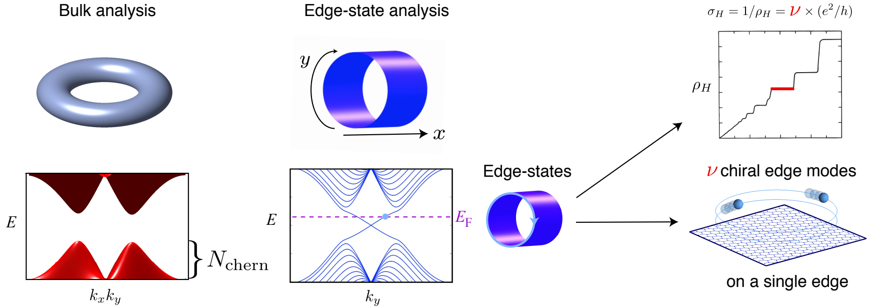
$\longrightarrow$ $\nu_{\rm exp} = 0.99(5)$

Ref: Aidelsburger, Lohse, Schweizer, Atala, Barreiro, Nascimbene, Cooper, Bloch, Goldman, Nature Phys. **11**, 162 (2015)

Seeing topological edge states with atoms



Bulk-Edge Correspondence in the Quantum Hall effect



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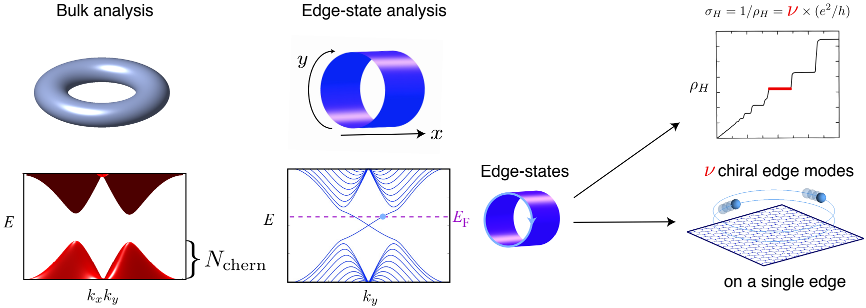


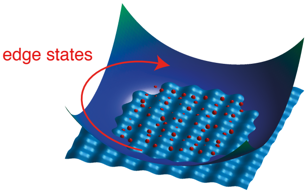
- Bulk-edge : the number of edge modes ν is *topologically protected*

$$\nu = N_{\text{chern}} \qquad\qquad \sigma_H = \frac{e^2}{h}\nu$$

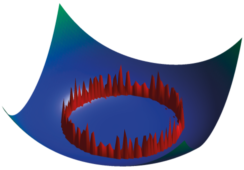
- Edge modes are 1D Dirac fermions : $E(k_y) \approx v k_y$
- The edge states chirality (orientation of propagation) : sign$(\partial E/\partial k_y) =$ sign(ν)

A quantum Hall device with cold atoms: *what's on the edge?*

Cold atoms in optical lattices:
emulating a QH insulator



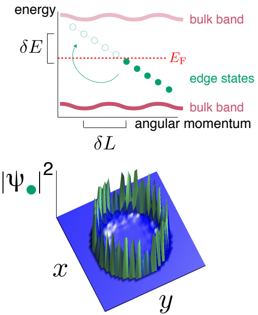
Goal: Isolating and seeing
the topological edge states



- How to recognize the edge states?
  - They are chiral ("*all go in the same direction*")
  - They are localized on the edge of the cloud
  - Their dispersion relation is linear: $E \sim vk$
- Main difficulty: many bulk states compared to only a very few edge states
  - Typically in a cloud: N=10.000 particles and about 10–100 edge states
  - **How to isolate the signal stemming from the edge states?**

Spectroscopy and atomic state manipulation

- Excite particles in the vicinity of the Fermi energy, i.e., in a topological bulk gap



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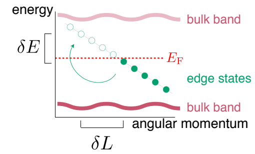
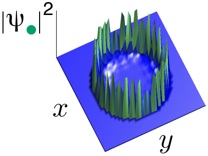
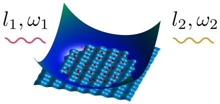
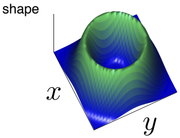




- The probe:
  two Laguerre-Gaussian beams



$$\delta\omega \approx \delta E/\hbar \quad \delta l \approx \delta L$$



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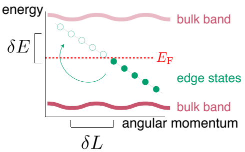

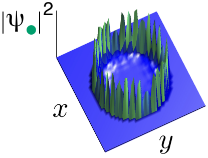

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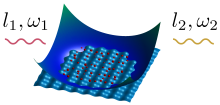

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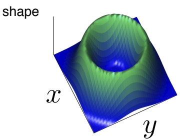
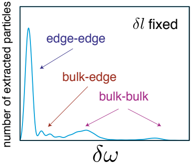
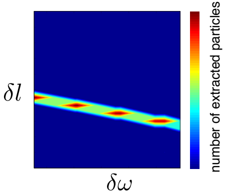


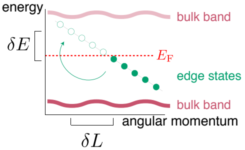
- The Bragg spectrum: revealing the dispersion relation

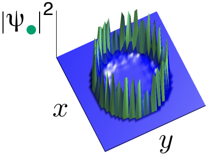




Spectroscopy and atomic state manipulation

- Excite particles in the vicinity of the Fermi energy, i.e., in a topological bulk gap





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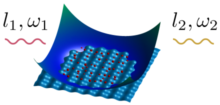

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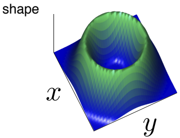

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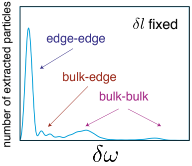
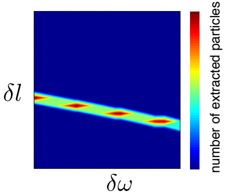
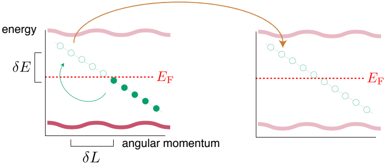




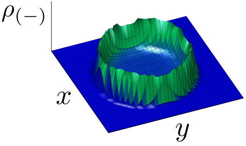
- Excite particles + change their internal state



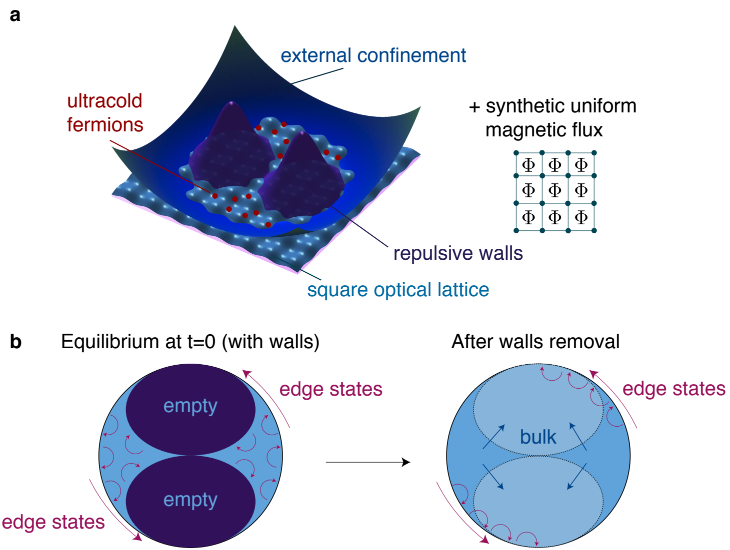
Occupied atomic state $|g_+\rangle$ Empty atomic state $|g_-\rangle$

- Edge-state signal on a dark background (without bulk)

N.G., J. Beugnon and F. Gerbier
PRL 2012

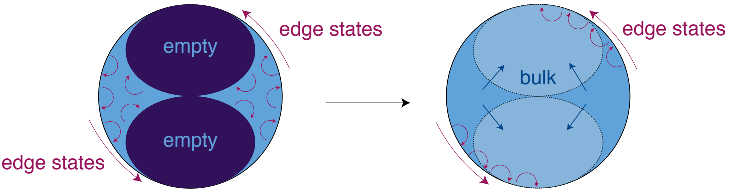


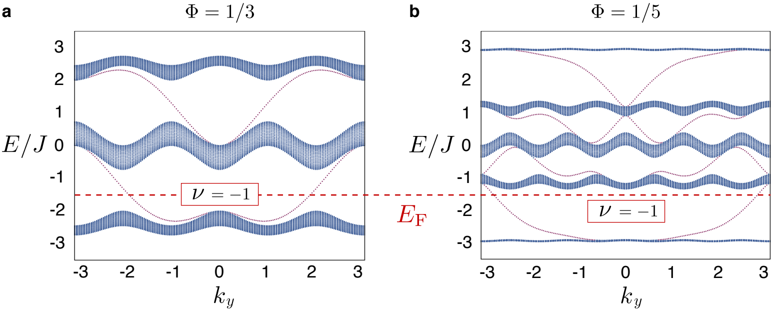
Probing the edge states after a quench : the bat geometry



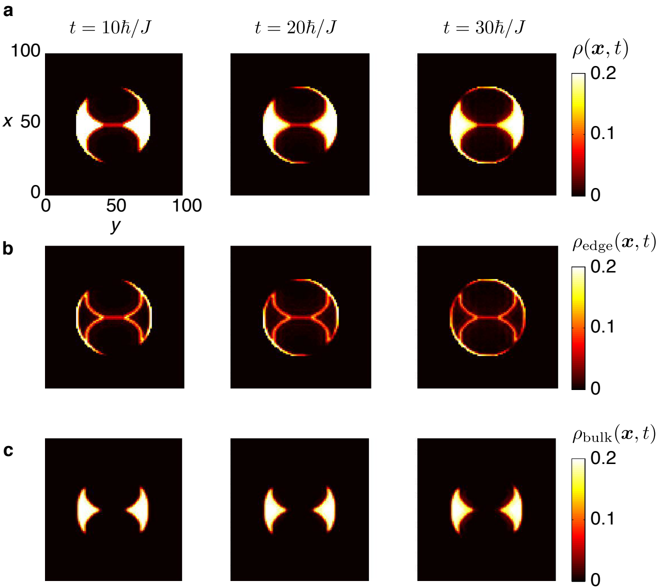
NG, J. Dalibard, A. Dauphin, F. Gerbier, M. Lewenstein, P. Zoller, and I. B. Spielman, PNAS 110, 6736 (2013)

Dispersive vs dispersionless systems

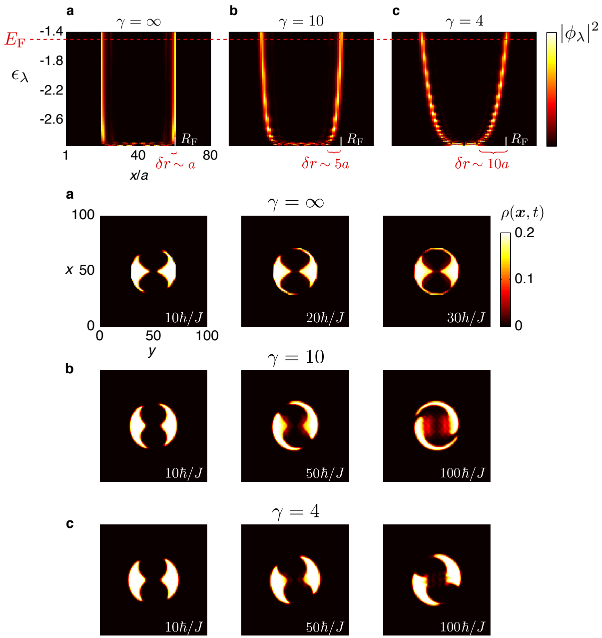




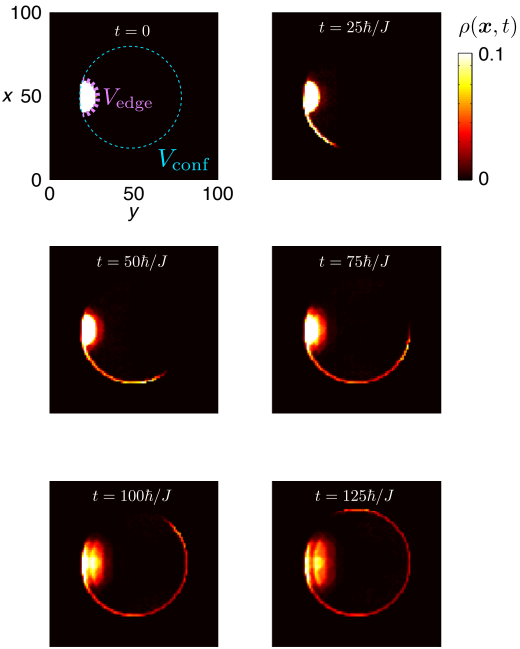
Dynamics for the topological flat band regime



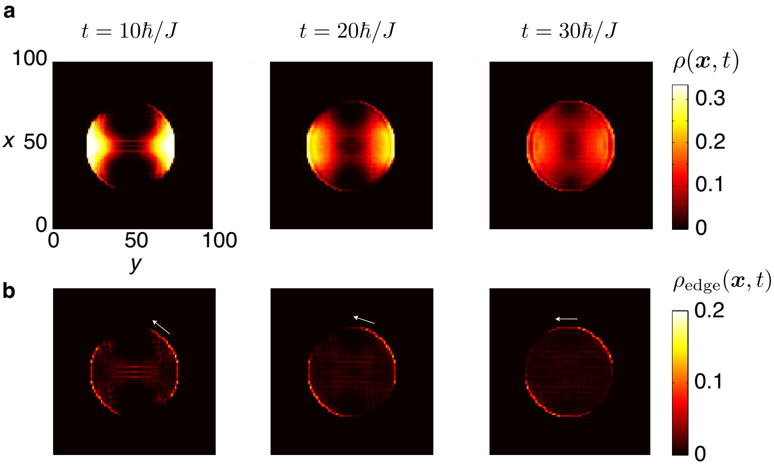
The effects of smooth confinements : $V(r) \sim (r/r_0)^\gamma$



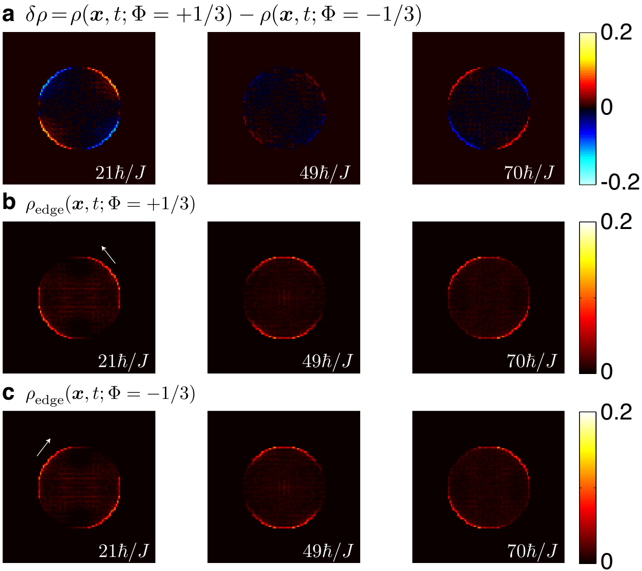
Squeezing the cloud against the edge



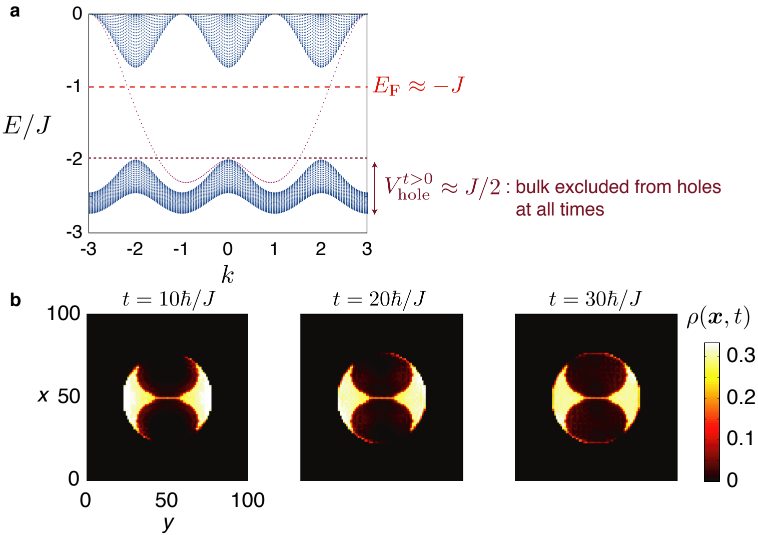
Dynamics for the dispersive system



The opposite flux method for dispersive systems



The edge-filter method for dispersive systems

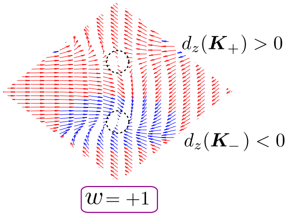


Some topics not discussed here

- Skyrmion-patterns in time-of-flight (Alba et al. PRL '11, Goldman et al. NJP '13)

$$H(\boldsymbol{k}) = \epsilon(\boldsymbol{k})\hat{1}_{2\times 2} + \boldsymbol{d}(\boldsymbol{k})\cdot\hat{\boldsymbol{\sigma}}$$: two-band systems (e.g. Haldane model)

$$N_{ch} = \frac{i}{2\pi}\int_{\mathbb{T}^2}\mathcal{F} = \frac{1}{4\pi}\int_{\mathbb{T}^2}\frac{\boldsymbol{d}}{d^3}\cdot\left(\partial_{k_x}\boldsymbol{d}\times\partial_{k_y}\boldsymbol{d}\right)\mathrm{d}^2\boldsymbol{k} = w,$$



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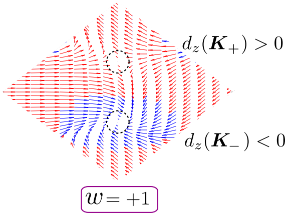

- Observation of chiral currents in optical ladders [see Part 3 and I. Bloch's lecture]
- Zak phase measurement [see I. Bloch's lecture]
- Berry curvature measurement through interferometry [see I. Bloch's lecture]
- Thouless pump realization [see I. Bloch's lecture]
- Proposal to probe Majorana edge modes in atomic wires [Kraus et al. NJP '12, Nascimbene JPB '13]

Outline

Part 1: Shaking atoms!

Generating effective Hamiltonians: “Floquet” engineering

Topological matter by shaking atoms

Some final remarks about energy scales

Part 2: Seeing topology in the lab!

Loading atoms into topological bands

Anomalous velocity and Chern-number measurements

Seeing topological edge states with atoms

Part 3: Using internal atomic states!

Cold Atoms = moving 2-level systems

Internal states in optical lattices: laser-induced tunneling

Synthetic dimensions: From 2D to 4D quantum Hall effects